PHY122 – Physics for the Life Sciences II

Lecture 7
Batteries, Capacitors, Resistors, and Currents

Note: Clicker Channel 41
Summary: Currents

Analogy: water flow = charge flow

= electrical current: \( I \equiv \frac{\Delta Q}{\Delta t} \)

- current flows from high electrical potential to lower potential
  - charge can be "pumped up" inside an emf ...
  - charge can be stored in a capacitor ...

Kirchhoff’s junction law:
- in an electrical junction, the total flow is conserved:
  - the current that goes in = the current that comes out
  - With convention in-going current is positive, out-going current is negative:

Kirchhoff’s loop law is equally intuitive:
- in any closed loop, the sum of all potential steps is zero ...
  \[ \sum_{i} \Delta V_i = 0 \]

Note: signs of \( \Delta V_i \) depend on the direction!
Resistance

**Resistance** \( R \) and Resistivity \( \rho \):

\[
R \equiv \frac{\Delta V}{\Delta I} = \frac{\Delta V}{I} = \rho \frac{L}{A}
\]

- The proportionality constant \( \rho \) is dependent on material, temperature, and typically not even constant with voltage...

- **Units:**
  - Resistance: \([R] = \text{V}/\text{A} \equiv \text{Ohm} = \Omega\)
  - Resistivity: \([\rho] = (\text{V}/\text{m})/(\text{A}/\text{m}^2) = \text{Vm/A} \equiv \text{Ohm}\cdot\text{m} = \Omega\text{m}\)

**Ohm's "Law":**

\[ \Delta V = IR \]

- **Combining Resistors:**
  - **Series:** \( \Delta V_{ab} = \Delta V_{ac} + \Delta V_{cb} \)
    \[ I R_{\text{equiv}} = IR_1 + IR_2 \Rightarrow R_{\text{equiv}} = R_1 + R_2 \]
  - **Parallel:** \( I_{\text{tot}} = I_1 + I_2 \)
    \[ \frac{\Delta V_{ab}}{R_{\text{equiv}}} = \frac{\Delta V_{ab}}{R_1} + \frac{\Delta V_{ab}}{R_2} \Rightarrow \frac{1}{R_{\text{equiv}}} = \frac{1}{R_1} + \frac{1}{R_2} \]
Percentage Body Fat …

can be measured by measuring the resistance of tissue because fat, muscle, and bone all have different resistivity: $\rho_f = 25 \ \Omega \cdot m$

$\rho_m = 13 \ \Omega \cdot m$

$\rho_{\text{bone}} \gg \rho_f$

Problem 22.66:
- A person’s leg measures $L = 40 \ \text{cm}$ between the knee and the hip, with an average leg diameter (ignoring bone and other poorly conducting tissue) of $D = 12 \ \text{cm}$. A potential difference of $\Delta V = 0.85 \ \text{V}$ causes a current of $I = 1.6 \ \text{mA}$. What is the percentage fat?
- Total resistance $R_{\text{tot}} = R_f \parallel R_m$:

$$\frac{1}{R_{\text{tot}}} = \frac{I}{\Delta V} = \frac{1}{R_f} + \frac{1}{R_m} = \frac{fA}{\rho_f L} + \frac{(1 - f)A}{\rho_m L}$$

$$= \frac{A}{L} \frac{\rho_m f + \rho_f (1 - f)}{\rho_f \rho_m}$$

$$= \frac{A}{L \rho_f \rho_m} \left( \left( \rho_m - \rho_f \right) f + \rho_f \right)$$

$$\frac{I}{\Delta V} \frac{L}{A} \rho_f \rho_m = \left( \rho_m - \rho_f \right) f + \rho_f \quad \Rightarrow \quad f = \left( \frac{I}{\Delta V} \frac{L}{A} \rho_m - 1 \right) \frac{\rho_f}{\left( \rho_m - \rho_f \right)} = 0.28$$

28% check units: OK!
What is the current (A) supplied by the battery in this circuit?

\[ I = \frac{V}{R_{tot}} \]

where
\[ R_{tot} = 5.0 + (10 \parallel 10) = 5.0 + 5.0 = 10 \, \Omega \]

\[ \Rightarrow I = \frac{20 \, V}{10 \, \Omega} = 2.0 \, A \]
HW Problem

Find the current through resistor a)

\[ I_a = \frac{V_\epsilon}{R_a + R_b \parallel (R_c + R_d)} = \frac{10 \text{ V} / \Omega}{5.0 + 10 \times 10/(10+10)} = 1.0 \text{ A} \]

Find the potential difference across resistor a)

\[ V_a = I_a R_a = 5.0 \text{ V} \]

Similarly:

Find the current through resistor b) and the potential difference across resistor b):

\[ V_b = V_\epsilon - V_a = 5.0 \text{ V} \]
\[ I_b = \frac{V_b}{R_b} = 5.0 \text{ V} / 10 \Omega = 0.50 \text{ A} \]

Find the current through resistor c) and the potential difference across resistor c)

\[ I_c = \frac{V_{cd}}{R_{cd}} = \frac{V_b}{(R_c + R_d)} = 5.0 \text{ V} / 10.0 \Omega = 0.50 \text{ A} \]
\[ V_c = I_c R_c = 2.5 \text{ V} \]
Capacitors in Series and Parallel

In electronics, capacitors are thin metal-foil/insulator stacks, often “rolled up” in a tiny cylindrical package with color-coded or printed values...

- **Common capacitor values:** \( C = 1 \text{ pF} - 1 \text{ mF} \) (±1%, 5%, 10%, …)

- **Symbols:**
  
  \[
  C \equiv \frac{Q}{\Delta V}
  \]

- **Combining capacitors:**

  • **Series:**
  \[
  \Delta V_{ab} = \Delta V_{ac} + \Delta V_{cb}
  \]
  
  \[
  \frac{Q}{C_{\text{equiv}}} = \frac{Q}{C_1} + \frac{Q}{C_2} \quad \Rightarrow \quad \frac{1}{C_{\text{equiv}}} = \frac{1}{C_1} + \frac{1}{C_2}
  \]

  • **Parallel:**
  \[
  Q_{\text{tot}} = Q_1 + Q_2
  \]
  
  \[
  C_{\text{equiv}}\Delta V_{ab} = C_1\Delta V_{ab} + C_2\Delta V_{ab} \quad \Rightarrow \quad C_{\text{equiv}} = C_1 + C_2
  \]
Energy and Power

The energy generated by the chemistry $E_{\text{chem}}$ in the battery is converted into electric potential energy $U_{\text{elec}}$...

When the battery is connected into a circuit, this potential energy is continuously converted into kinetic energy of motion $K$ of the charges circulating in the circuit:

The kinetic energy is continuously converted into heat (i.e. $E_{\text{th}}$) in the wires and resistors as the charges accelerate and bump into the conductor atoms and loose most of their energy...

inside capacitors, the energy may get stored ...

\[
P = \frac{|\Delta U|}{\Delta t}
\]

Units: W(att) ≡ J/s

\[
\text{Power delivered by a battery (emf } \mathcal{E}): \quad P_{\mathcal{E}} = I_{\mathcal{E}} \mathcal{E} \quad (W=VA)
\]

\[
\text{Power dissipated as heat in a resistor:} \quad P_R = I_R \Delta V_R = I_R^2 R
\]

\[
\text{Power going into storage on a charging capacitor:} \quad P_C \equiv \frac{\Delta U_C}{\Delta t} = \frac{1}{2} \frac{dQ_C^2}{C \, dt} = \frac{1}{2} \frac{2Q_C \, dQ_C}{C \, dt} = I_C \Delta V_C
\]
What is the power (W) supplied by the battery in this circuit?

\[ P = \frac{V_{\text{bat}} I_{\text{bat}}}{R_{\text{tot}}} = \frac{V_{\text{bat}}}{R_{\text{tot}}} \times V_{\text{bat}}, \text{ where} \]
\[ R_{\text{tot}} = 5.0 + (10 \ || 10) \]
\[ = 5.0 + 5.0 \]
\[ = 10 \ \Omega \]
\[ \Rightarrow P = \frac{20^2}{10} = 40 \text{ W} \]
HW Problem

Calculate the equivalent capacitance:

\[ C_1=3 \text{ } \mu\text{F}, \quad C_2=10 \text{ } \mu\text{F}, \quad C_3=3 \text{ } \mu\text{F}, \quad C_4=5 \text{ } \mu\text{F}, \quad C_5=6 \text{ } \mu\text{F}, \quad C_6=5 \text{ } \mu\text{F} \]

\[ C_2 \parallel C_6 = ? = \left( \frac{1}{10} + \frac{1}{5} \right)^{-1} = \frac{10}{3} = 3.33 \text{ } \mu\text{F} \]

\[ C_3 + C_5 + C_4 = ? = (3 + 6 + 5) \text{ } \mu\text{F} = 14 \text{ } \mu\text{F} \]

\[ C_3 + C_5 + C_4 + C_2 \parallel C_6 = ? = (3 + 6 + 5 + 3.33) \text{ } \mu\text{F} = 17.33 \text{ } \mu\text{F} \]

\[ C_1 \parallel (\cdots) = ? = \left( \frac{1}{3} + \frac{1}{17.33} \right)^{-1} = \cdots \]
Which of the following combinations of capacitors has the highest capacitance?

A. A
B. B
C. C
D. D

\[ C_A = C \]
\[ C_B = C \parallel C = 2C \]
\[ C_C = (C^{-1} + C^{-1})^{-1} = C/2 \]
\[ C_D = (C^{-1} + (2C)^{-1})^{-1} = 2C/3 \]

\[ \Rightarrow B \]
Batteries and Meters

The ideal emf delivers ANY amount of current, while keeping the terminal voltage constant

- **IDEAL** emf:  $r_i = 0$
- A **real** emf:  often:  $r_i \approx 1 \Omega$

Ideal volt meters measure the voltage, without drawing ANY current

- **IDEAL** V-meter:  $R_i = \infty$
- A **real** voltmeter:  Resistance:  $R_i \geq 1 \text{ M}\Omega$

Ideal Ammeters have zero resistance (zero voltage drop)

- **IDEAL** A-meter:  $R_{\text{SHUNT}} = 0$
- A **real** ammeter:  Resistance:  $R_{\text{SHUNT}} \leq 100 \Omega$
HW Problem

Consider the circuit shown.

All wires are considered ideal;
Resistance $R$ is constant;
the internal resistances of the battery $r$ and the ammeter $r_A$ are non-zero,
the internal resistance of the voltmeter $r_V$ is not infinite.

Q: What is the reading $V$ of the voltmeter?
Express your answer in terms of the resistances and $\mathcal{E}$.

$$R_{eq} = r + r_V \parallel (r_A + R) \Rightarrow I_\mathcal{E} = \frac{\mathcal{E}}{R_{eq}} \Rightarrow V_r = I_\mathcal{E} r$$

$$\Rightarrow V = \mathcal{E} - V_r$$

Note: $V_r$ is a voltage DROP, and subtracts from the battery’s emf $\mathcal{E}$

Q: What is the reading $I_A$ of the ammeter?

$$\Rightarrow I_A = \frac{V}{(R + r_A)} = \cdots$$

NOTE: $R \parallel r \equiv \frac{Rr}{R + r}$
Connecting a charged capacitor to a resistor gets current to flow, which slowly decreases until the capacitor is fully drained:

After switch $S$ is closed ($t=0$):

$$V(t) - IR = 0 = \frac{Q(t)}{C} + R \frac{dQ}{dt} \Rightarrow \frac{dQ}{dt} = -\frac{1}{RC}Q(t) \Rightarrow Q(t) = Q_0 e^{\frac{t}{RC}}$$

And equivalently:

$$V(t) = \frac{Q(t)}{C} = \frac{Q_0}{C} e^{-\frac{t}{RC}} = V_0 e^{-\frac{t}{RC}}$$

$$I(t) = \frac{V(t)}{R} = \frac{V_0}{R} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{\tau}}$$

Note the CHARACTERISTIC TIME $\tau = RC$:
- after $t=RC$, charge, current, and voltage are down to $1/e$,
- after $t=2RC$, they are down to $1/e^2$, etc.

Note the changed sign: $V(t)$ is $+ve$; $Q(t)$ diminishes with time, so the slope $dQ/dt < 0$ !

Generally: whenever the rate of change in a quantity is proportional to the quantity itself, we get exponential behavior!

E.g.: population explosion!

$Q(t)$ decreases over time, reaching $Q_0/e$ after $RC$.
All capacitors are initially charged to 5.0 V. All the switches are closed at $t=0$; which capacitor has the highest voltage at $t=1$ s?

A. A  B. B  C. C  D. D

Question is equivalent to asking which circuit has the largest $RC$ value, because that circuit will discharge slowest!

$\Rightarrow$ C
- Initially, the capacitor is uncharged;
- at time $t=0$ the switch is closed
- Kirchhoff’s Loop Rule in this circuit:

\[
E - \frac{Q}{C} - IR = 0 \quad \Rightarrow \quad V_0 - \frac{Q}{C} - R \frac{dQ}{dt} = 0
\]

\[
\frac{CV_0 - Q}{C} = R \frac{dQ}{dt} \quad \Rightarrow \quad dt = \frac{RC}{CV_0 - Q} dQ
\]

\[
\Rightarrow \quad t = \int_0^t dt = \int_0^{Q} \frac{RCdQ}{CV_0 - Q} = -RC \int_{CV_0}^{Q} dx = -RC \ln \frac{CV_0 - Q}{CV_0}
\]

\[
\Rightarrow \quad CV_0 - Q = CV_0 e^{-\frac{t}{RC}} \quad \Rightarrow \quad Q(t) = CV_0 \left(1 - e^{-\frac{t}{RC}}\right) = CV_0 \left(1 - e^{-\frac{t}{\tau}}\right)
\]

- Equivalently: the Voltage across the capacitor:

\[
V(t) = \frac{Q(t)}{C} = V_0 \left(1 - e^{-\frac{t}{\tau}}\right)
\]
In a series $\mathcal{E}RC$ circuit ($C$ is uncharged before the switch is closed) ...

A. The current is smallest immediately after the switch is closed ...

B. The voltage across the capacitor is highest immediately after the switch is closed ...

C. The voltage across the capacitor is highest long after the switch is closed ...