PHY122 – Physics for the Life Sciences II

Lecture 12
Faraday’s Law of Induction

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Like dielectric materials in electric fields, materials often have magnetic properties as well, characterized by the relative magnetic permeability $K_m$ multiplying $\mu_0$:

- Thus: $\mu_0 \rightarrow \mu \equiv K_m\mu_0$ in the formulae!

We distinguish three categories:

- **Paramagnetic** materials: 
  intrinsic atomic magnetic moments (electron orbital and electron intrinsic spin) align with the external magnetic field; alignment is only approximate as thermal “jitter” disturbs alignment:
  \[ K_m \approx 1.0001 - 1.001 \] (small)

- **Diamagnetic** materials: 
  some materials without intrinsic magnetic moment can acquire an INDUCED magnetic moment (rotating charge distributions that are pulled apart) which anti-aligns with the external magnetic field:
  \[ K_m \approx 0.998 - 0.999 \] (small)

- **Ferromagnetic** materials: 
  in materials like Fe, local “domains” of mutually aligned intrinsic spins arise spontaneously. Even a moderate external field will align these domains, which gives an enormous boost to the magnetic field:
  \[ K_m \approx 1000 - 10^5 \] (very large)
Origin of Intrinsic $\mu$

Electrons orbit inside atoms (classical, incorrect picture!)
- a circulating current $\Rightarrow$ orbital magnetic moment $\mu$:

$$I_e = \frac{q}{T\text{(period)}} = \frac{-e}{2\pi R/\nu_e} = \frac{-ev_e}{2\pi R} \quad \mu_{\text{orbital}} = IA = \frac{-ev_e}{2\pi R} \pi R^2 \hat{\mathbf{A}} = \frac{-ev_e R}{2} \hat{\mathbf{A}}$$

- using angular momentum $|\mathbf{L}| = |\mathbf{R} \times \mathbf{p}| = mv_e R$: $\mu_{\text{orbital}} = \frac{-ev_e R}{2} \hat{\mathbf{A}} = \frac{-e}{2m} \mathbf{L}_{\text{orbital}}$

The electron also “spins” (this is a true “quantum” effect, but it behaves analogous to a spinning top):
- experimentally:

$$\mu_{\text{spin}} = 2 \frac{-e}{2m} \mathbf{L}_{\text{spin}}$$

Quantum Mechanics: any angular momentum $L$ is quantized:

$$L = n \frac{\hbar}{2\pi}, \text{ with } n=0,1,2,\ldots \text{ an integer!}$$
- Only noticeable for atomic systems because $\hbar$ (Planck’s constant) is extremely small: $\hbar = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$
Applications: The Electric Motor

- A current loop in an external (uniform or symmetric) magnetic field experiences a net torque but no net force:

\[ F_{\text{top}} = ILB \sin \theta_{L,B} = ILB \sin 90^\circ = ILB = F_{\text{bottom}} \quad \Rightarrow \quad F_{\text{top}} = -F_{\text{bottom}} \]

\[ F_{\text{front}} = IHB \sin \theta_{H,B} = IHB \sin (90^\circ - \theta) = IHB \cos \theta = F_{\text{back}} \quad \Rightarrow \quad F_{\text{front}} = -F_{\text{back}} \]

Torque:
\[ \tau = \left| \sum_i R_i \times F_{Ri} \right| = 2 \frac{H}{2} F_{\text{top}} \sin 90^\circ = HF_{\text{top}} \sin \theta = I \left( LH \right) B \sin \theta \]

\[ \tau = IAB \sin \theta \]

with Magnetic moment of the loop \( \mu = IA \): \[ \tau = \mu \times B \]

In a motor, an **alternator** ensures that the current switches sign whenever the axis \( A \) is (anti-)parallel to \( B \) …
Applications: the Loudspeaker

- The loudspeaker coil receives AC current $I$ representing the (amplified) sound.

- the coil is immersed in a transverse magnetic field from a strong permanent magnet (heavy!)

- the Magnetic force is then up or down, depending on the current’s direction ...  
  • Right-Hand-Rule!
  • Magnitude:

$$F_B = ILB \sin \theta_{L,B} = I (2\pi R) B \sin 90^\circ = I (2\pi R) B$$

- Thus, the coil plus the loudspeaker cone above, vibrate up and down and produce sound waves in the surrounding air ...
Chapter 25 - Overview

• Magnetic Induction – a changing magnetic flux induces an emf:
  – motional emf
  – magnetic Flux
    • definition …
    • induced emf by a changing flux
    • Faraday’s Law
    • Direction of induced emf: Lenz’ Law

• Applications:
  – the Generator

• Electromagnetic Waves
  – The propagation of self-sustaining electric and magnetic field oscillations
  – Intensity
  – The EM spectrum
  – Photons
Michael Farday’s Experiments

After the discovery of the magnetic interaction from currents (moving charges) Faraday sought to find the “reverse”:

- Can magnetism create currents?
- Result: yes, **CHANGING** magnetic fluxes can induce electric currents! (Note: currents \(\Rightarrow\) emf !)
- **FLUX**: has to do with the AMOUNT of magnetic field lines “seen” by the current circuit …

Note: these motions are RELATIVE motions
Faraday’s Induction Law

Consider a \( L \) m long piece of conducting rod moving with velocity \( \mathbf{v} \) perpendicular to a uniform and constant magnetic field \( \mathbf{B} \):

- the Lorentz force on the charge carriers \( q \) in the rod is:

\[
\mathbf{F}_L = q \mathbf{v} \times \mathbf{B} = qvB \uparrow \text{(up)}
\]

- the \( \mathbf{F}_L \) pumps the charges UP (R-H-R) ...
- they accumulate at the ends of the rod: +ve at the top, -ve on the bottom, until an equilibrium is established with the electric field ...

an \textbf{emf} is created:

\[
\overline{E} = \Delta V = -EL = -q \frac{\mathbf{F}_L}{L} = -qvB
\]

\[
= -B \frac{dA}{dt} = -B \frac{d(BA)}{dt}
\]

\( BA \) is the "Magnetic Flux"

- where \( A \) is the area covered by the rod per period \( \Delta t \)
- the -ve sign indicates the \textbf{emf} opposes the change in flux (Lenz’ Law)
Magnetic Flux $\Phi_B$

**Magnetic Flux $\Phi_B$** is defined as the “amount of field seen by the circuit”

- Mathematically: $\Phi_B \equiv \int B \cdot dA \approx \mathbf{B} \cdot \mathbf{A} = BA \cos \theta_{B,A}$
- units: $T \cdot m^2 \equiv \text{Wb (Weber)}$

- Graphically:

  loop seen from the side

  loop as seen looking towards the magnetic field

  New “effective area” $A_{\text{eff}} = a \times b \cos \theta = A \cos \theta$
Electromotive Force due to \textit{Changing} Flux

Experiments (Faraday et al.) have shown that:

\textit{CHANGES IN MAGNETIC FLUX} $\Phi_B$ \textit{over a surface area cause an INDUCED EMF in the curve bounding the area}

\textbf{Faraday's Law:} \[ \mathcal{E} = \int_{\text{bounding curve}} \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt} = EL \approx -\frac{\Delta \Phi_B}{\Delta t} \]

- with: $\Phi_B \approx \mathbf{B} \cdot \mathbf{A} = BA \cos \theta_{B,A}$

- Faraday's Induction Law:
  \[ \mathcal{E} = -\frac{d\Phi_B}{dt} = -\frac{d(BA \cos \theta_{B,A})}{dt} \]

Thus, a \textbf{changing magnetic flux} can result from:

- a change in field $B$: i.e. $dB/dt \neq 0$
- a change in area $A$: i.e. $dA/dt \neq 0$
- and/or a change \textit{in the angle between} $\mathbf{B}$ \textit{and} $\mathbf{A}$: i.e. $d\cos \theta_{A,B}/dt \neq 0$

We'll consider examples of each of these in the following...
Changing Magnetic Flux ➔ EMF

Consider a loop of \( N \) windings and area \( A \) immersed in a uniform magnetic field \( B \), making an angle of \( \theta = 30^\circ \) with the normal to the plane of the loop.

Assume the field is changing over time as: \( B = B_0 e^{-t/\tau} \) (e.g. due to a charging capacitor) with \( \tau \) some characteristic time constant:

- Note the field is DECREASING with time:

\[
\frac{dB}{dt} = \frac{B_0}{\tau} e^{-t/\tau} \leq 0 \quad \text{and} \quad \mathcal{E} = \frac{d\Phi_B}{dt} = \frac{B_0 NA \cos \theta}{\tau} e^{-t/\tau}
\]

The meaning of the -sign in \( \mathcal{E} = -d\Phi_B / dt \)

- the induced emf is such that it opposes the change in flux;
  - i.e. the resulting current in the loop will be \( I_{\text{induced}} = \mathcal{E} / R \),
  - and the induced current will flow in such a direction that it opposes the change in flux (a decrease for this example)
- i.e. here it will be such that \( B_{\text{induced}} \) strengthens the instantaneous field which is decreasing with time...

This is Lenz' Law
Generator: Changing the Flux Area $A$

The slide-wire generator:

- Force $F_{\text{ext}}$ pulls the rod with constant $v$.
- The area of the loop changes (increases): $dA/dt = L \, dx/dt = L \, v$
- As before, the Lorentz force $F_L$ pushes the charge carriers up in the rod.
- From where they flow around the loop to the bottom of the rod:
  $$|\mathcal{E}| = d\Phi_B / dt = B \, dA / dt = B \, d(Lx) / dt = B L v$$

The flux $\Phi_B = B_{\text{ext}} \cdot A$ is increasing here; indeed the induced current $I_{\text{ind}} = E / R = B L v / R$ produces a $B_{\text{induced}}$ OUT of the page,

- In accordance with Lenz’ Law!
  - Note that the induced current interacts with $B_{\text{ext}}$ to give a force to the LEFT, opposite and equal to $F_{\text{ext}}$ (if not, we would get free energy!)
- Dissipated power: $P_{\text{dis}} = I_{\text{ind}}^2 R = B^2 L^2 v^2 / R$
- Applied power: $P_{\text{ext}} = F_{\text{ext}} \cdot v = -F_{\text{opposing}} v = I_{\text{ind}}(\uparrow) L B_{\text{ext}} v = B_{\text{ext}}^2 L^2 v^2 / R$

$\Rightarrow$ Lenz’ Law ensures Energy Conservation...
Generators: Changing the Angle between A, B

Consider a static and uniform $\mathbf{B}$-field to the right. A wire loop ($N$ windings, area $\mathbf{A}$) rotates around an axis perpendicular to $\mathbf{B}$ (perpendicular to the paper), with constant angular velocity $\omega$:

- Thus only the relative orientation of $\mathbf{B}$ and $\mathbf{A}$ varies with time...
- The change in flux:

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(\mathbf{B} \cdot \mathbf{A}) = BA \frac{d \cos \theta_{AB}}{dt} = BA \frac{d \cos(\omega t)}{dt} = -\omega BA \sin(\omega t)$$

$$\Rightarrow \mathcal{E} = -N \frac{d\Phi_B}{dt} = \omega NBA \sin(\omega t)$$

- i.e. induced EMF varies sinusoidally in time, with amplitude $\omega NBA$
- Numerical example: an AC generator:

$f = 60$ Hz $= \omega/2\pi$, $N = 500$, $A = 3.0 \text{ m}^2$, $B = 0.10 \text{ T}$

$$\mathcal{E} = 2\pi f NBA = 120\pi \times 500 \times 3.0 \times 0.10 = 57 \text{ kV},$$

i.e. good for High Voltage power line distribution
Using Lenz’ Law, determine the direction of the induced current in circuit 2 \((a \rightarrow b\) or \(b \rightarrow a)\) when:

- with switch \(S\) closed, coil 2 is moved CLOSER to coil 1:
  - coil 2 moves into a \textit{stronger} \(B\) field (\(B\) to \textit{right}) \(\Rightarrow\) flux in coil 2 \textit{increases}.
  - Lenz’ Law: current induced in 2 \textit{opposes} this \(\Rightarrow\) induced field to the \textit{left}; i.e. induced current in 2: \(a \rightarrow b\)

- keeping the switch closed, resistance \(R\) is \textit{reduced}:
  - Field (to \textit{right}) from coil 1 becomes \textit{stronger} \(\Rightarrow\) flux in coil 2 \textit{increases}.
  - Lenz’ Law: current induced in 2 \textit{opposes} this \(\Rightarrow\) induced field to the \textit{left}; i.e. induced current in 2: \(a \rightarrow b\)

- after having been closed, switch \(S\) is now \textit{opened}:
  - when switch opens: \(B\) field (\(B\) to \textit{right}) suddenly \textit{decreases} \(\Rightarrow\) flux in 2 \textit{decreases}.
  - Lenz’ Law: current induced in 2 \textit{opposes} this \(\Rightarrow\) induced field to the \textit{right}; i.e. induced current in 2: \(b \rightarrow a\)
Problems
Problem

A long wire carrying a 4.0 A current perpendicular to the xy-plane intersects the x-axis at $x = -2.0$ cm. A second, parallel wire carrying a 3.0 A current intersects the x-axis at $x = +2.0$ cm.

- At what point on the x-axis is the magnetic field zero if the two currents are in the same direction?
Problem

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- At what point on the $x$-axis is the magnetic field zero if the two currents are in the same direction?

The field of two wires: $\vec{B} = \vec{B}_1 + \vec{B}_2 \overset{?}{\Rightarrow} 0$

$\vec{B}_1 = \frac{\mu_0}{2\pi} \frac{I_1}{2-x} \overset{?}{\Rightarrow} 0$

$\vec{B}_2 = \frac{\mu_0}{2\pi} \frac{I_2}{2+x} \overset{?}{\Rightarrow} 0$

\[ \frac{\mu_0}{2\pi} \frac{I_1}{2-x} \overset{?}{\Rightarrow} \frac{\mu_0}{2\pi} \frac{I_2}{2+x} \]

\[ I_1 (2+x) = (2-x)I_2 \quad (\text{x in cm!}) \quad \text{units ok} \]

\[ 3(2+x) = (2-x)4 \]

\[ 6 + 3x = 8 - 4x \Rightarrow 7x = 2 \Rightarrow x = \frac{2}{7} \text{ cm} \quad \checkmark \]

makes sense: P is closer to $I_1$ ($I_1 < I_2$)
Mass Spectrometer

A mass spectrometer is designed to separate protein fragments.

The fragments are ionized by removing a single electron and then enter a $B=0.80\ T$ uniform magnetic field at a speed of $v=2.0\times10^5\ m/s$.

- If a fragment has a mass that is 85 times the mass of the proton ($m_p=1.67\times10^{-27}\ kg$), what will be the distance between the points where the ion enters and exits the magnetic field?
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\[
\overrightarrow{F}_B = e \overrightarrow{v} \times \overrightarrow{B} \implies F_B = e v B \sin(90^\circ) = m_p c = m r^2
\]

\[
\implies r = \frac{m v}{e B} \implies 2r = \frac{2 m v}{e B} = \frac{170 m_p v}{e B} = ...
\]
A device called a *rail gun* uses the magnetic force on currents to launch projectiles at very high speeds.

A 1.5 V power supply is connected to two conducting rails. A segment of copper wire, in a region of uniform magnetic field, slides freely on the rails.

The wire has a 1.0 m$\Omega$ resistance and a mass of 5.0 g. Ignore the resistance of the rails.

- Calculate the wire's speed after it has slid a distance of 10.0 cm?
Rail Gun

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- Calculate the wire's speed after it has slid a distance of 10.0 cm?

\[ \vec{F}_B = I \vec{v} \times \vec{B} \Rightarrow \vec{F}_B = I \vec{v} \times B \sin(90°) \text{ to the right} \]

\[ I = \frac{V}{R} \Rightarrow \vec{F}_B = ma = \frac{V}{R} \vec{v} \times B \]

\[ \Rightarrow a = \frac{\vec{v} \times B}{mR} = \ldots \]

\[ v_x^2 = v_{0x}^2 + 2ax \quad (\text{PHY 121!}) \Rightarrow v_x = \sqrt{2 \frac{\vec{v} \times B}{mR}} x = \ldots \]
Levitating a Current-Carrying Wire

A copper wire of radius $R=0.50$ mm and length $L$ carries a current $I=50.0$ A to the East.

We apply to this wire a magnetic field $B$ that produces on it an upward force exactly equal in magnitude to the wire's weight ($\rho_{\text{Cu}}=8920$ kg/m), causing the wire to "levitate."

- Calculate the required field (direction and magnitude) ...
A copper wire of radius \( R = 0.50 \text{ mm} \) and length \( L \) carries a current \( I = 50.0 \text{ A} \) to the East.

We apply to this wire a magnetic field \( \mathbf{B} \) that produces on it an upward force exactly equal in magnitude to the wire's weight \( (\rho_{\text{Cu}} = 8920 \text{ kg/m}) \), causing the wire to "levitate."

- Calculate the required field (direction and magnitude) ...

\[ F_B = I L B, \text{ and direction up } \uparrow \]
\[ \Rightarrow \mathbf{B} \text{ is directed into paper!} \quad (\text{North}) \]

\[ F_B = I L B = W = m g = \rho_{\text{Cu}} V g = \rho_{\text{Cu}} (\pi R^2 L) g \]

\[ \Rightarrow B = \frac{\rho_{\text{Cu}} \pi R^2 g}{I} = \ldots \]
Additional Questions

A bar magnet sits inside a coil of wire that is connected to a meter. The bar magnet is at rest in the coil. What can we say about the current in the meter?

A. The current goes from right to left.
B. The current goes from left to right.
C. There is no current in the meter.

![Diagram of a bar magnet inside a coil of wire connected to a meter]
A bar magnet sits inside a coil of wire that is connected to a meter. The bar magnet is pulled out of the coil. What can we say about the current in the meter?

A. The current goes from right to left.
B. The current goes from left to right.
C. There is no current in the meter.

the magnet is pulled out; the B field is to the left and the flux through the solenoid is diminishing → induced current tries to strengthen the left-directed field → $I_{\text{induced}}$ goes from right to left through the meter
Additional Questions

A bar magnet sits inside a coil of wire that is connected to a meter. The bar magnet is completely out of the coil and at rest. What can we say about the current in the meter?

A. The current goes from right to left.
B. The current goes from left to right.
C. There is no current in the meter.
A bar magnet sits inside a coil of wire that is connected to a meter. The bar magnet is reinserted into the coil. What can we say about the current in the meter?

A. The current goes from right to left.
B. The current goes from left to right.
C. There is no current in the meter.

The magnet is inserted; the field is to the left and the flux through the solenoid is increasing. The induced current tries to oppose the rise in the flux. \( I_{\text{induced}} \) goes from left to right through the meter.
Additional Example Problems

Two metal loops face each other. The upper loop is suspended by plastic springs and can move up or down. The lower loop is fixed in place and is attached to a battery and a switch. Immediately after the switch is closed,

A. Is there a force on the upper loop? If so, in which direction will it move? Explain your reasoning.

B. Is there a torque on the upper loop? If so, which way will it rotate? Explain your reasoning.

When the switch is closed, the counterclockwise current produces a \( B \) field that is up and the flux through the upper loop is increasing \( \Rightarrow \) the induced current tries to oppose this increase and therefore is clockwise \( \Rightarrow I_{\text{induced}} \) interacts with the bottom loop to repel each other. This repulsion is the same all around the loop, and therefore there is no torque (apart from the asymmetry caused by the battery & switch side-loop).