PHY122 – Physics for the Life Sciences II

Lecture 13
AC-Circuits
Capacitors and Inductors

Note: Clicker Channel 41
Chapter 26 – AC Electricity

• **AC Voltage Sources**
• **AC Resistor circuits**
  - *AC* currents in a resistor
  - *AC* power and RMS voltage and current
• **AC Capacitor circuits**
  - *AC* voltage and current in a capacitor
  - Reactance of a capacitor
• **Inductors**
  - *AC* voltage and current in an inductor
  - Self Inductance and Reactance
• **Transformers**
  - Electrical Power in your home and Electrical safety
• **AC circuits with R, C, and L**
  - ERLC circuits
AC Voltage Sources

An AC Generator:
- A wire loop is rotated by an external force (water or hot gas turbine) in an magnetic field:
- The Lorentz force $F_L = qv \times B = qvB \sin \theta$ on the mobile charges in the moving wire creates an induced current $I_{ind}$:
- Faraday’s induction law: $\mathcal{E} = -\frac{d\Phi_B}{dt}$; $I_{ind} = \mathcal{E}/R$
- The voltage is picked up by sliding contacts at the two ends of the wire loop ...

Real Generator:
- many windings
- multiple magnet poles
AC Generator: Changing Angle between Area and B

Consider a static and uniform B-field to the right.

A wire loop ($N$ windings, area $A$) rotates around an axis perpendicular to $B$ (perpendicular to the paper), with constant angular velocity $\omega$:

- Here, only the relative orientation of $B$ and $A$ varies with time...
- The change in flux:

$$\frac{d\Phi_B}{dt} = \frac{d}{dt}(B \cdot A) = BA \frac{d\cos \theta_{AB}}{dt} = BA \frac{d\cos(\omega t)}{dt} = -\omega BA \sin(\omega t)$$

$$\Rightarrow \quad \mathcal{E} = -N \frac{d\Phi_B}{dt} = \omega NBA \sin(\omega t) = \mathcal{E}_0 \sin(\omega t)$$

- induced emf varies (co)sinusoidally in time,
  - with amplitude $\mathcal{E}_0 = \omega NBA$
- Example: a large AC generator:

$f = 60$ Hz$=\omega/2\pi$, $N=500$, $A=3.0$ m$^2$, $B=0.10$ T

$\Rightarrow \quad \mathcal{E}_0 = 2\pi f NBA = 120\pi \times 500 \times 3.0 \times 0.10 = 57$ kV,
  i.e. good for High Voltage power line distribution
The **Angular Frequency** of the emf in US household outlets is ... 

\[ \omega = 2\pi f = 2\pi (60 \text{ s}^{-1}) = 377 \text{ rad/s} \]
AC Circuits…

After discussing AC circuits with resistors only, we are now interested in going one step further and discuss Alternating Current (AC) circuits with Capacitors.

In AC circuits, elements that are dependent on current CHANGES, will play important roles, e.g.:

- Capacitors: \( V_C = \frac{q}{C} = \int idt/C \)
- Coils or inductors: \( V_L = -L \frac{di}{dt} \) (we will discuss this in the next slides)

Before embarking on the AC project, we discuss the relationship between current and voltage in the elements \( R \), \( C \), and \( L \) in greater detail.
Current and Voltage in Resistors

An AC current $i_R(t) = I_R \cos\omega t$ flows through resistor $R$.

The current through the resistor follows Ohm's Law $V=IR$.

The current creates a $\Delta V$ between the resistor's terminals:

- a voltage DROP in the direction of the current
- voltage difference: $v_R(t) = R i_R(t) = R I_R \cos(\omega t) = V_R \cos(\omega t)$

Observations:

- The voltage and current are IN “SYNC”: the voltage is maximum when the current is maximum ($\omega t=n\pi$, $n=1,2,3,...$), and vice versa!
- in RESISTOR: VOLTAGE IN SYNC WITH CURRENT, i.e. the voltage varies proportionally with the current...

- the IMPEDANCE, i.e. the
- (equivalent resistance):
  \[ X_R \equiv V/I = R \ (\Omega) \]  

\[ \omega \equiv d\theta/dt = 2\pi f \]
I connect a 60 W lamp to a 120 V outlet; the current in the lamp is ...

\[ I = \frac{V}{R} = \frac{VI}{IR} = \frac{P}{V} = \frac{60 \text{ W}}{120 \text{ V}} = 0.5 \text{ A} \]
AC Power in Resistors

Power is the product of the *instantaneous* current and voltage: \( P(t) = v(t) \cdot i(t) \):

- for a resistor we found: \( v(t) = R I_R \cos(\omega t) \) \( i(t) = I_R \cos(\omega t) \)
- Thus, the power dissipated in the resistor \( R \) equals:
  \[
P(t) = v(t) \cdot i(t) = R I_R^2 \cos^2(\omega t) = P_0 \cos^2(\omega t)
  \]
- this oscillates also, but is always \( \geq 0 \);
  However, the interesting measure is the AVERAGE power (that is what we pay for!):
  \[
  \overline{P}(t) = R I_R^2 \cos^2(\omega t) = R \frac{I_R^2}{2} = R \left( \frac{I_R}{\sqrt{2}} \right)^2 = R \left( I_{\text{rms}} \right)^2
  \]
- where: \( I_{\text{rms}} \equiv \frac{I_R}{\sqrt{2}} \) and similarly: \( V_{\text{rms}} \equiv \frac{V_R}{\sqrt{2}} \) (rms = “root-mean-square”)
- combining: \( \Rightarrow \overline{P} = V_R I_R \frac{1}{2} = V_{\text{rms}} I_{\text{rms}} \)
Problem

A toaster oven is rated at $P=1400$ W for operation at $V_{\text{rms}}=120$ V/60 (Hz).

a) What is the resistance $R$ of the oven heater element?

$$\bar{P} = 1400 \text{ W} = V_{\text{rms}} I_{\text{rms}} = V_{\text{rms}} \frac{V_{\text{rms}}}{R} = \frac{V_{\text{rms}}^2}{R} \quad \Rightarrow \quad R = \frac{V_{\text{rms}}^2}{P} = \cdots$$

b) What is the peak current $I_{\text{peak}}$ through the oven?

$$\bar{P} = 1400 \text{ W} = V_{\text{rms}} I_{\text{rms}} \quad \Rightarrow \quad I_{\text{peak}} = I_{\text{rms}} \sqrt{2} = \frac{P}{V_{\text{rms}}} \sqrt{2} = \cdots$$

c) What is the peak power $P_{\text{peak}}$ dissipated by the oven?

$$P_{\text{peak}} = \ ? = 2\bar{P}$$
The following devices are all plugged into the same 120 V circuit in a house. This circuit is protected with a 10 A circuit breaker. Will the circuit blow?

<table>
<thead>
<tr>
<th>Device</th>
<th>Power (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Computer</td>
<td>120 W</td>
</tr>
<tr>
<td>Stereo Amplifier</td>
<td>180 W</td>
</tr>
<tr>
<td>Electric Heater</td>
<td>900 W</td>
</tr>
<tr>
<td>Lamp</td>
<td>100 W</td>
</tr>
</tbody>
</table>

\[
P_{\text{total}} = 1300 \text{ W} = VI = 120I \Rightarrow I > 10 \text{ A}
\]

A. Yes
B. No
C. Abstain ...
Current and Voltage in *Capacitors*

Take a oscillatory current $i(t) = I \cos \omega t$ to a capacitor $C$.

The (dis)charging current creates a voltage difference between the capacitor’s plates...

- voltage difference: $v_C(t) = \frac{q(t)}{C} = \frac{1}{C} \int i_c dt = \frac{1}{\omega C} I C \sin \omega t = +V_C \sin \omega t$

Observations:

- Voltage and current are OUT OF SYNC, the current is maximum at $T/4 = 90^\circ$ BEFORE the voltage:

- i.e. there is a PHASE difference of $90^\circ$, or: the voltage and current are $90^\circ$ out of phase...

- the **IMPEDANCE** (or **REACTANCE**): $X_C \equiv \frac{V}{I} = \frac{1}{\omega C}$ (Ω)
Inductance: emf and Sources of B

A changing magnetic flux induces an emf: a changing flux $d\Phi_{B,2}/dt$ over wire loop 2 creates a current in the loop, which opposes the flux change. A magnetic field might be created by a current (e.g. in a solenoid), and a changing current $i_1$ creates a changing magnetic field $\mathbf{B}$. The proportionality factor between $di_1/dt$ and $\mathcal{E}_2$ is the MUTUAL INDUCTANCE $M_{12}$ between the coils: $\mathcal{E}_2 = -M_{12} \frac{di_1}{dt}$.

- $M$ specifies the effect ($\mathcal{E}_2$) in a second conductor due a change of current $i_1$ in the first, mediated by $\mathbf{B}_1$. Minus sign: $\mathcal{E}_2$ opposes the change in flux ...

- Typically, $M_{12}$ depends ONLY on geometrical factors
  - (size, relative orientation, #windings) of the coils...

- Unit of Inductance: H(enry) $\equiv V/(A/s) = \Omega \cdot s$
Self Inductance

Whereas Mutual Inductance describes the effect of a changing current in one coil on the \textit{emf} in another coil, \textit{there is also an induced emf in the first coil ITSELF!}

- That is (or should be) completely expected:
  - a coil carrying a \textit{changing current}, itself experiences the changing flux as well, and thus should itself experience a \textit{self-induced emf}...

- the \textit{self-induced} emf implies the existence of a \textit{voltage difference} $\Delta V$ between the terminals of the coil ...
  - that opposes the original current change ...

Taking the definition of \textit{mutual inductance}, we can easily define the \textbf{SELF-INDUCTANCE} $L$:

$$\mathcal{E}_1 = \Delta V_{\text{coil}} \equiv -L \frac{di_1}{dt} \quad \text{(Note again the "symbolic" - sign!)}$$

where $\mathcal{E}_1$ is the \textit{self-induced} \textit{emf} due to the changing current $i_1$...
The current in a 0.20 H inductor changes (decreases) by –500 Ampere per second; the voltage drop across the inductor equals:

\[ \Delta V = \mathcal{E} = L \frac{di}{dt} = 0.20 \text{ H} \times \frac{500 \text{ A}}{\text{s}} = 100 \text{ V} \]
Calculating Self-Inductance

Self-inductance of a long solenoid (cross section \( A \), \( N \) windings, length \( l \)):

\[
B = \begin{cases} 
\mu_0 \frac{Ni}{l} & \text{inside and near the center of the long solenoid} \\
\approx 0 & \text{outside an idealized long solenoid}
\end{cases}
\]

\[
\Rightarrow -E = N \frac{d\Phi_B}{dt} = N \frac{d(BA)}{dt} = NA \frac{dB}{dt}
\]

\[
= NA \frac{\mu_0 N}{l} \frac{di}{dt} = L \frac{di}{dt} \quad \Rightarrow \quad L_{\text{solenoid}} = \frac{\mu_0 AN^2}{l}
\]