Lecture 22
Quantum Physics - the Heisenberg Uncertainty Principle

Note: Clicker Channel 41
Recap: Electron Diffraction

A beam of “monochromatic” electrons “shining” on a double slit, produces an interference pattern!

- **EVEN if they come 1 electron per second !!**

BUT: **ONLY if we do not** measure WHICH of the two slits they go through!

- Aahh, the magic of quantum physics!
- deduced from the size $d$ of the slit, the momentum $p=mv$ of the electrons, and the interference pattern:

\[ d \sin \theta_m = n\lambda; \quad n = 0, \pm 1, \pm 2, \pm 3, \ldots \]

\[ \lambda = \frac{h}{p} \]

\[ hc = 1240 \text{ eV} \cdot \text{nm} \]

remember: 1 eV = 1 e J/C = 1.6\times10^{-19} \text{ J}
What is “Waving” for a Particle ??

A lot of discussion arose about the question of what exactly is “waving” in a “particle wave”;

- in an EM wave it is the \( \mathbf{E} \) and \( \mathbf{B} \) field vectors that are oscillating …

how to interpret the wave function associated with particles?

- A point-like particle at position \( x \) represented by a wave ??

Try: a localized particle represented by a “wavelet” \( \psi(x) \):

consequences:

- exact wavelength is uncertain: \( \Delta \lambda \approx \frac{\lambda}{n_{\text{waves}}} \)
- thus, momentum has uncertainty:

\[
\Delta p_x = \Delta \left( \frac{h}{\lambda} \right) = \frac{h}{\lambda^2} \Delta \lambda = \frac{h}{\lambda n_{\text{waves}}}
\]

- exact position is uncertain: \( \Delta x \approx n_{\text{waves}} \lambda \)
- more waves (longer wavelet): \( \Delta x \uparrow \) and \( \Delta p_x \downarrow \)

- PRODUCT CONSTANT: \( \Delta x \Delta p_x \approx n_{\text{waves}} \lambda \times h/(n_{\text{waves}} \lambda) = h \)
- in fact, this is a minimum: Heisenberg’s Uncertainty Relationship:

\[
\Delta x \Delta p_x \geq \frac{h}{4\pi}
\]

\( \rightarrow \psi \) uncertain

\( \rightarrow p_x \) uncertain

\( \lambda \) uncertain

\( \text{and } x \text{ uncertain!} \)
Heisenberg’s Uncertainty Principle

Direct consequence of the Wave Character of particles; and a fundamental property of Quantum Physics!

- NOT simply a limitation due to instrument measurement error!

Consider another example:

- diffraction of an electron beam in a small slit
- like for light, a diffraction pattern appears
  - pattern is MUCH wider than the slit!

- calculate:
  - for the electron momentum \( p_y \): \( p_y = \frac{h}{\lambda} \)
  - 1\textsuperscript{st} diffraction minimum when:

\[
\theta_{\text{min}} \approx \frac{w/2}{L} = \frac{\lambda}{a} \quad \Rightarrow \quad \frac{\lambda}{a} = \frac{w/2}{L} = \frac{\Delta p_x}{p_y} = \frac{\Delta p_x}{h/\lambda}
\]

\[\Rightarrow a \Delta p_x = h \quad \Rightarrow \quad \Delta x \Delta p_x = h\]

Impossible to predict where any given diffracted electron will hit the screen; most probably somewhere in the central maximum …
Still: What is Waving??

The question still remains: what is waving?

**Interpretation:**

- analogue: oscillating E field ⇒ but what we actually “see” on the screen is the **INTENSITY**: \( I \propto E^2 \)
- the particle “is” the wave function \( \psi(x) \);
- but what we “see” is the **square** \( |\psi(x)|^2 \)!
  - \( |\psi(x)|^2 \) is the **PROBABILITY** of finding the particle at position \( x \)!
  - (requires “normalization”: total probability must be 100%: \( \int |\psi(x)|^2 \, dx = 1 \) )

other **Uncertainty Relationships** can be derived:

a very important one: \[ \Delta \tau \Delta E \geq \frac{h}{2\pi} \]

- a relation between the uncertainties in the “lifetime” and in the “energy” (mass) of a particle, an atomic or nuclear state, ....
- all are based on the “wave-character” of particles at atomic/nuclear scales!
Bound Particles and Energy Quantization

When a particle is “confined” ...
- e.g. an electron attached to an atom
- or a particle in a “box” of length $L$

... it will produce

**STANDING (PROBABILITY) WAVES**!
- exactly like waves on a drum,
- ... or on a fixed string of length $L$!
- but: it is $\sqrt{\text{probability}}$ that waves!

**Standing Wave** condition:
\[ L = n \frac{\lambda_n}{2}; \quad n = 1, 2, 3, \ldots \quad \Rightarrow \quad \lambda_n = \frac{2L}{n} \]

Because $p = \frac{h}{\lambda}$:
\[ p_n = \frac{h}{\lambda_n} = n \frac{h}{2L}; \quad n = 1, 2, 3, \ldots \]

Hence, both **momentum** $p$ and (kinetic) **energy** $E$ are QUANTIZED:
\[ E_n = \frac{1}{2} mv^2 = \frac{p_n^2}{2m} = n^2 \frac{h^2}{8mL^2} = n^2 E_1; \quad n = 1, 2, 3, \ldots \]
Particle in a Box

consider again an electron in a box of size $L$:

$$L = n \frac{\lambda_n}{2}; \quad n = 1, 2, 3, \ldots \quad \Rightarrow \quad \lambda_n = \frac{2L}{n}$$

$n=1$: Blue wavefunction - - - - -

probability $|\psi(x)|^2$ 

largest probability for particle to be near the center

$n=2$: Red wavefunction - - - - -

probability $|\psi(x)|^2$ 

largest probability for particle to be near middle of left or right halves!

$n=29$: Black wavefunction - - - - -

probability $|\psi(x)|^2$ 

~flat probability for particle to be anywhere in the box $\Rightarrow$ CLASSICAL

From Quantum to Classical regime for large $n$!
Example

An electron (mass \( m = 511 \text{ keV}/c^2 \)) is confined in a "box" of \( L = 0.194 \text{ nm} \).

- calculate the **energy levels** of the system:

\[
E_n = n^2 \frac{\hbar^2}{8mL^2} = n^2 \frac{(hc)^2}{8mc^2 L^2} ; \quad n = 1, 2, 3, \ldots
\]

\[
= n^2 \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8 \times 511 \text{ keV} \times (0.194 \text{ nm})^2}
\]

\[
= n^2 (10.0 \text{ eV})
\]

- a **DISCRETE energy spectrum**:

These are allowed energies. The electron cannot have an energy between these.
Transitions between Energy Levels

A confined electron thus may have only discrete energies ... The electron can “jump” from one level to another ...

- to a **HIGHER** energy level by **ABSORPTION** of a photon of precisely the right energy,
- to a **LOWER** energy level by **EMISSION** of a photon with energy equal to the energy level difference

This explains the behavior of atomic systems:

- the emission and absorption of photons of discrete wavelengths!

\[ E_y = \frac{hc}{\lambda} = \Delta E = |E_{n_1} - E_{n_2}|; \quad n_1, n_2 = 1, 2, 3, \ldots \]

\[ \Delta E_{\text{system}} = |E_3 - E_1| = 80 \text{ eV} \]
\[ \Delta E_{\text{system}} = |E_1 - E_2| = 30 \text{ eV} \]

A transition from \( n = 1 \) to \( n = 3 \) requires the absorption of an 80 eV photon.
A transition from \( n = 2 \) to \( n = 1 \) requires the emission of a 30 eV photon.
Atomic Emission and Absorption Spectra

This explains the behavior of atomic systems:
- the emission and absorption of precise wavelengths!
- Helium emission spectrum:
- Fraunhofer absorption lines in the Solar Spectrum:
- Hydrogen, Helium, ...
Quantum Tunneling

in classical mechanics a rolling ball with a certain (kinetic) energy $K = \frac{p^2}{2m}$ can never cross a hill higher than a certain height $h$ given by $mgh = K$:

- heights $> h$ (and POTENTIALS $V > mgh$) are “forbidden” territory!

NOT true in quantum physics!

- while boxes with INFINITE walls cannot be escaped from,
- boxes with FINITE walls are escapable!
- the wave function of energy $K < V$ penetrates somewhat into the “classically forbidden” region:
  - Hence, if the potential wall is “thin” enough, the wave function will “leak out” on the far side!!
  - meaning: the particle escapes (with some small probability)!
  - explains, for instance, $\alpha$-RADIOACTIVITY!
  - AGAIN: perfectly OK for light WAVES to cross a small gap (“frustrated total internal reflection”)
Applications of Quantum Physics

MANY Examples:

• Selective activation of substances
• Selective detection & identification of substances
• Tunneling – Scanning Tunneling Microscope
• Atomic Force Microscope...
• Nuclear Magnetic Resonance (Imaging)
• Lasers
• X-ray generation
• Modern ultra-fast electronics

• Chemistry ...
• Micro-biology ...
Photons:
- light QUANTA with energy \( E=hf=hc/\lambda \), with \( hc=1240 \text{ eV} \cdot \text{nm} \)
- proof from the Photoelectric Effect

(sub)Atomic particles exhibit wave character:
- momentum \( \Leftrightarrow \) wavelength: \( p=h/\lambda \)
- the particle system has an associated wave function \( \psi(r) \)
- wave function squared \( |\psi(r)|^2 \) describes the probability for finding a particle at position \( r \).

Consequences:
- mono-energetic light and particle beams both exhibit INTERFERENCE and diffraction ...
- when CONFINED (e.g. in box of length \( L \)), the system shows DISCRETE energy/momentum levels
  - e.g. for particle in a box: \( E_n=n^2h^2/(8mL^2) \)
  - transitions between energy states via emission and absorption of photons of energy equal to the energy level difference
  - going from the quantum to the classical regime: \( n \to \text{large} \)